gian
GLOBAL INITIATIVE OF ACADEMIC NETWORKS

सत्यमेव जयते
Ministry of Education
Government of India

GLOBAL INITIATIVE OF ACADEMIC NETWORKS
(GIAN)

9th June to 19th June, 2025

IIT Indore, Khandwa Road, Simrol

Indore 452020, Madhya Pradesh

Solving linear systems and computing generalized inverses using recurrent neural networks

भारतीय प्रौद्योगिकी संस्थान इंदौर
Indian Institute of Technology Indore
IIT Indore
॥ ज्ञानम् सर्वजनहिताय ॥

Overview

The aim of this course is to collect the latest developments in the theory and computation in numerical linear algebra using algorithms based on dynamical systems. The main topics included in this course are computing the usual matrix inverse and generalized inverses, solving systems of linear matrix equations and linear systems. Both static and dynamic problems are considered. he proposed topic is an introduction and overview of the relatively new research field in Numerical Linear Algebra.

The dynamical system approach is a powerful tool for solving many kinds of matrix algebra problems because of:
(a) possibility to ensure a response within a predefined time in real-time applications,
(b) its parallel distributed nature,
(c) convenience of hardware implementation,
(d) global convergence without conditions,
(e) in addition, dynamical system approach is applicable to online computation with time-varying matrices.

Two main dynamical systems are considered as appearances of Recurrent Neural Networks: Gradient Neural Networks (GNN) and Zeroing Neural Network (ZNN). GNNs are designed essentially for constant (or static, time-invariant) coefficient matrices and vectors. Objects involved in Zeroing Neural Network (ZNN) dynamics are time-varying. The ZNN method was proposed in 2001. ZNN dynamical evolution is based on an indefinite error function and an appropriate system of differential equations used to achieve exponential error decay.

ZNN methods help in optimizing and controlling the behavior of robots and autonomous vehicles, localization of moving objects, and other. On the other hand, generalized inverses are powerful tools and are applicable in many branches of mathematics, technics, and engineering. The most common and important application is in finding solutions to many matrix equations and systems of linear equations. In addition to numerical linear algebra, there are many other mathematical and technical disciplines in which generalized inverses play an important role. Some of them are estimation theory (regression), computing polar decomposition, electrical circuits (networks) theory, automatic control theory, filtering, difference equations, pattern recognition, image restoration.

Our primary goal is to apply dynamical systems approach in numerical linear algebra, especially in the computation of generalized inverses and solving system of linear matrix equations. Many classes of generalized inverses have been proposed and investigated so far. The most popular are the Moore-Penrose inverse and the Drazin inverse. Computation models are defined in terms of continuous state space dynamical systems that appear as recurrent neural networks phenomena. In general, algorithms focused on continuous-time computation for solving time-invariant and time-varying problems in real and complex domain are considered. There is an increasing interest in continuous-time computation, where the states of a dynamical system evolve continuously. Many biological systems and some control systems can be better described in the analog manner. The main motivation is the goal to derive efficient models for the variety of continuous dynamical systems that are present in the world.

Therefore, time-varying matrix computations are a new, separate and still unexplored area of numerical linear algebra that is worth studying and learning.

Modules	**A: Introduction and basic GNN models** **B: Basic ZNN models** **C: Advanced GNN and ZNN models** **Number of participants for the course will be limited to fifty.**
Module A **Introduction and basic GNN models**	**Day1** **Lecture 1: 1 hr (PS)** Short introduction to matrix inversion and generalized matrix inversion. Particularly, introduction of the Moore-Penrose, weighted Moore-Penrose, Drazin and Group inverse, {i,j,k} inverses, inner inverses, and outer inverses with prescribed range and null space. Least squares solutions and best approximate solution. **Lecture 2: 1 hr (PS)** Definitions of gradient and Hessian, matrix and vector norms. Basic principles and methods in nonlinear unconstrained optimization, overview of line search methods. Overview of gradient-descent methods, Newton method and quasi-Newton methods, conjugate gradient nonlinear optimization methods. **Lecture 3: 1 hr (SSA)** Matrix decompositions, Singular Value Decomposition (SVD), Moore Penrose Inverse, computation of condition numbers of a matrix. **Day 2** **Lecture 4: 1 hr (PS)** Recurrent Neural Networks (RNN), Continuous-time RNN, Gradient Neural Networks (GNN), GNN dynamics for solving linear matrix equations AXB=C, GNN for computing generalized inverses of constant matrices, GNN for solving systems of linear equations. **Tutorial 1: 2 hr (PS)** Simulink as an efficient tool for agile software development. Simulink implementation of GNN for solving the general linear matrix equations AXB=C. Simulink for GNN design for computing the matrix inverse, left and right inverse and the Moore-Penrose generalized inverse. Simulink implementation of GNN models for solving systems of linear equations. **Lecture 5: 1 hr (SSA)** Computation of SVD, computation of condition numbers, solving least square problems using SVD. **Day 3** **Lecture 6: 1 hr (PS)** Design parameters in GNN evolutionary design. Properties of activation functions in RNN, overview of commonly used activation functions (AFs): linear, bipolar sigmoid, power AF, power-sigmoid, hyperbolic sine, sign-bi-power, tunable AF. Influence of gain parameters and activations functions on the convergence speed. **Lecture 7: 1 hr (SSA)** LU and its Sensitivity analysis. Pivoting, matrix rank, matrix range and null space. **Tutorial 2: 2 hrs (SSA)** Computations of LU and solving systems of equations using forward and backward substitutions.
Module B **Basic ZNN models**	**Day 4** **Lecture 8: 1 hr (PS)** Basic principles of Zhang Neural Networks (ZNN), scalar-valued, vector-valued and matrix-valued error function for time-varying inversion. **Lecture 9: 1 hr (PS)** Development of ZNN models for solving time-varying scalar-valued reciprocal, vector-valued models for solving system of linear equations, matrix-valued and matrix-valued time-varying inversion. ZNN for solving over-determined and under-determined systems of linear equations. **Tutorial 3: 2 hrs (PS)** Implementation of nonlinear GNN and ZNN models based on various activation functions. Numerical experiments and comparison of obtained results regarding influence of the gain parameter and AFs.

	Day 5 **Lecture 10: 1 hr (PS)** Solving time-varying inversion problems using scalar-valued, vector-valued and matrix-valued error functions. Computing time-varying generalized inverses. **Lecture 11: 1 hr (SSA)** QR matrix decomposition and its application to solve least square problems, minimum norm least square solution. **Tutorial 4: 2 hrs (SSA)** Computation of Reflectors, rotators, to solve least squares problem using QR factorizations. **Day 6** **Lecture 12: 1 hr (PS)** Application of ZNN in solving time-varying linear matrix equations AXB=C and systems of linear equations Ax=b. **Lecture 13: 1 hr (PS)** Application of ZNN in solving time-varying linear matrix equations AXB=C, Lyapunov equation AX+XAT+Q=0, Sylvester equation AX+XB=C and non-symmetric algebraic Riccati equation DX+XA-XBX+Q=0. **Tutorial 5: 2 hrs (PS)** Implementation of ZNN models for solving various time-varying matrix equations. Applications in computing generalized inverses and solving linear systems.
Module C Advanced GNN and ZNN models	**Day 7** **Lecture 14: 1 hr (PS)** Correlations between GNN and gradient descent methods of unconstrained nonlinear optimization. Newton iteration for matrix inversion as discretized ZNN continuous-time dynamics. Modifications of GNN and ZNN dynamics arising from gradient and Newton optimization methods. **Lecture 15: 1 hr (PS)** Survey about nonlinear activations. Overview of main activation functions which enable finite time and predefined time convergence. Finite-time and predefined-time convergence analysis of ZNN design. **Lecture 16: 1 hr (PS)** Development of ZNN models for solving scalar-valued and matrix-valued time-varying inversion and generalized inversion. **Day 8** **Lecture 17: 1 hr (PS)** Implementation of finite-time ZNN models. Experiments on finite time convergent ZNN dynamical systems and comparison of various activations. **Lecture 18: 1 hr (PS)** Application of ZNN in approximating time-varying square root, inverse square root, constrained matrix equations and various matrix functions. **Lecture 19: 1 hr (PS)** Overview of activation functions in RNN, survey on nonlinear activations. Finite-time and predefined-time convergent ZNN based on nonlinear AFs. Finite-time ZNN design. **Day 9** **Lecture 20: 1 hr (PS)** Existence and representations of solutions to some constrained matrix equations and systems of matrix equations. Computation of various generalized inverses arising from corresponding systems of matrix equations. **Tutorial 6: 2 hrs (SSA)** Computation of Eigenvalues by using Raleigh Quotient shifts and Wilkinson Shifts. **Lecture 21: 1 hr (PS)** Modified ZNN dynamical systems, ZNN based on optimization method, ZNN models based on iterative methods, ZNN dynamical systems based on fuzzy and neutrosophic logic systems

Day 10

Lecture 22: 1 hr (PS)
RNNs for solving linear inequalities and equations, RNNs for generating LU decomposition, QR decomposition and Cholesky factorization.
Lecture 23: 1 hr (SSA)
QR-Algorithm for computing Eigenvalues.
Lecture 24: 1 hr (PS)
Applications of ZNN neural design in mobile object localization, time-varying nonlinear equations solving.
Discussion about possibilities for further research.

Who can attend?

- Students at all levels (BTech/MTech/BSC/MSc/PhD) or Faculty from reputed academic institutions and technical institutions.
- Mathematics and engineering graduate students, practitioners in engineering, researchers in the areas of optimization, dynamical systems, numerical linear algebra, control systems, signal processing, simulation and modeling, analog hardware, and robotics.
- Researchers and students in applied mathematics, statistics, engineering, and many other scientific disciplines including research laboratories from academic and technical institutions.

Fees

Registration

The course is conducted in offline mode.

The participation fees for taking the course are as follows:

Undergraduate/Postgraduate : INR 1500 + 18 % GST= INR 1770
Ph.D.+ Post-Docs: 2000 + 18 % GST = INR 2360
Faculty: INR 4000 + 18 % GST = INR 4720
Industry person: INR 8000 + 18 % GST = INR 9440
Foreign participants: $ 200 + 18 % GST= $ 236

Last date to register: 15th May 2025

The above fee includes all instructional materials, computer use for tutorials and assignments, laboratory equipment usage charges, 24 hr free Internet facility. The participants will be provided with accommodation on payment basis. For any questions, please send an email to giangnn25@iiti.ac.in

The accommodation charges for the workshop are INR 392/- + bedroll & hygiene charges.
Total charges will be INR 510/- per day per person and the accommodation will be given on a shared basis single room with double occupancy.

How to Register:

Interested candidates can fill the form for which the link is given below:

https://docs.google.com/forms/d/e/1FAIpQLSfkPdVQ_necvmi9ck0OA3o93MB5s7boCVO_Mn9s2GtQjmqvZA/viewform?vc=0&c=0&w=1&flr=0&pli=1

Please click on the link to complete the payment requested by the merchant.

https://payu.in/web/EB3AF4CBC22FB4C90B5ABC9A52E5CAC3

Teaching Faculty

Prof. Predrag S. Stanimirović received his doctorate in computer science from the University of Niš, Faculty of Philosophy, Niš, Serbia. He is a full professor at the Faculty of Sciences and Mathematics, University of Niš, Department of Computer Science, Niš, Serbia. He gained thirty-six years of experience in scientific research in various fields of mathematics and computing, which include several branches of numerical linear algebra, recurrent neural networks, linear algebra, symbolic computation, nonlinear optimization, linear programming, and others. His main research topics include numerical linear algebra, operations research, recurrent neural networks, and symbolic computation. He has successfully published over 330 publications in scientific journals, including 5 research monographs, 6 textbooks, 5 monographs and over 80 peer-reviewed research articles published in proceedings and book chapters. He is the editor of the scientific journals Electronic Research Archive (ERA), Filomat, Facta Universitatis, Series: Mathematics and Informatics, and several other journals. The author is on Stanford's list of 2% top scientists in the world in 2021, 2022, and 2023.

ORCID ID: 0000-0003-0655-3741
Scopus ID: 390188
Web of Science Researcher ID: AAW-3979-2021
Website homepage: https://www.pmf.ni.ac.rs/nastavnici-i-saradnici/biografija/?idz=59

Course Co-ordinator

Prof. Ahmad completed his M.Sc. in Mathematics from Utkal University. He then pursued an M.Phil. from Ravenshaw University. In 2008, he was awarded a Ph.D. in Mathematics from IIT Guwahati. Following the submission of his thesis, Dr. Ahmad joined the Supercomputer Education and Research Centre (SERC) at IISc Bangalore as a Research Associate. In February 2009, he visited the Institut für Mathematik at the Universität Berlin, Germany, as a Postdoctoral Fellow. In December 2009, Dr. Ahmad returned to India and took up the position of Assistant Professor in the Discipline of Mathematics at IIT Indore. He was promoted to Associate Professor in 2016. During his tenure at IIT Indore, he served as Faculty Coordinator from 2010 to 2012 and as Head of the Department from 2016 to 2018. Dr. Ahmad's research interests include Numerical Linear Algebra, Quaternion and Reduced Biquaternion Linear Algebra, Matrix Polynomials, Multiparameter Eigenvalue Problems, and various Inverse Eigenvalue Problems. He is actively involved in several projects funded by the Department of Science and Technology (DST), Government of India. Additionally, he has visited several international institutions for research programs under various schemes of the Science and Engineering Research Board (SERB).

Duration:

9th June to 19th June, 2025

Course Co-ordinator

Prof. Sk. Safique Ahmad
Professor
Department of Mathematics
IIT Indore, Khandwa Road, Simrol
Indore 452020, Madhya Pradesh
Tel: +919691229996
Email: safique@iiti.ac.in
https://www.iiti.ac.in/people/~safique/